Homework Set #1.

Due Date: Wednesday January 16, 2019

1. Prove via the dual correspondence definition that the hermitian conjugate of $|\alpha><\beta|$ is $|\beta><\alpha|$. [1 point]

2. Prove that, in the absence of degeneracy, a sufficient condition for the following to be true

$$\sum_{b'} < c'|b' >< b'|a' >< a'|b' >< b'|c' >=$$

$$= \sum_{b',b''} < c'|b' >< b'|a' >< a'|b'' >< b''|c' >$$

(where as usual $|a' >$ is an eigen-ket of A etc.) is that $[A, B] = 0$ or that $[B, C] = 0$. [2 points]

3. Show that for the $|S_z; + >$ state of a spin $\frac{1}{2}$ system $< S_x^2 > - < S_x >^2 = \hbar^2/4$. [2 points]

4. A two-state system is characterized by the Hamiltonian

$$H = H_{11}|1 >< 1| + H_{22}|2 >< 2| + H_{12}[|1 >< 2| + |2 >< 1|]$$

where H_{11}, H_{12}, and H_{22} are real numbers with the dimension of energy, and $|1 >$ and $|2 >$ are eigenkets of some observable $\neq H$. Find the energy eigenkets and corresponding energy eigenvalues. [3 points]

5. Construct the transformation matrix that connects the S_z diagonal basis to the S_x diagonal basis and show that your result is consistent with the general relation

$$U = \sum_r |b^{(r)} >< a^{(r)}|.$$

[2 points]

6. An operator A, corresponding to an observable α, has two normalized eigenstates $|\phi_1>$ and $|\phi_2>$, with eigenvalues a_1 and a_2. An operator B, corresponding to an observable β, has normalized eigenstates $|\chi_1>$ and $|\chi_2>$. The eigenstates are related by

$$|\phi_1>=\frac{2|\chi_1>+3|\chi_2>}{\sqrt{13}}, \qquad |\phi_2>=\frac{3|\chi_1>-2|\chi_2>}{\sqrt{13}}.$$

α is measured and the value a_1 is obtained. If β is then measured and then α again, find the probability of obtaining a_1 a second time.

[5 points]