Homework Set #3.

Due Date: Wednesday January 30, 2019

1. For a certain system, the operator corresponding to the physical quantity A does not commute with the Hamiltonian. It has eigenvalues a_1 and a_2 corresponding to the eigenstates

$$|\phi_1>=(|u_1>+|u_2>)/\sqrt{2}, \quad |\phi_2>=(|u_1>-|u_2>)/\sqrt{2}$$

   where $|u_1>$ and $|u_2>$ are eigenstates of the Hamiltonian with eigenvalues E_1 and E_2. If the system is in the state $|\psi>=|\phi_1>$ at $t=0$, calculate the expectation value of A at time t. [3 points]

2. At time $t=0$ the wave function of a free particle in a one-dimensional system is

$$\psi(x,;0)=c\exp(-x^2/4\Delta_0^2),$$

   with c and Δ_0 constants. Show that Δ_t, the uncertainty in position at time t, is given by

$$\Delta_t^2=\Delta_0^2+(\Delta v)^2t^2,$$

   where Δv is the uncertainty in the velocity at $t=0$. How does the uncertainty in velocity vary with time? [5 points]

3. Consider a spin-$\frac{1}{2}$ particle with magnetic moment $\vec{\boldsymbol{\mu}}=\gamma\vec{\boldsymbol{S}}$. At time $t=0$, we measure S_y and find a value of $+\frac{1}{2}\hbar$ for its eigenvalue. Immediately after this measurement, we apply a uniform time-dependent magnetic field parallel to the z-axis. The B-field is chosen such that the Hamiltonian is:

$$H(t)=\omega_o(t)S_z\,,$$

   where

$$\omega_o(t)=\begin{cases}0\,, & \text{for } t<0\,,\\ \dfrac{\omega_o t}{T}\,, & \text{for } 0\le t\le T\,,\\ 0\,, & \text{for } t>T\,.\end{cases}$$

(a) Write down the time-dependent Schrodinger equation that governs the time evolution of the spin-$\frac{1}{2}$ particle of this problem.

(b) Solve the differential equation(s) obtained in part (a). Show that at time t, the particle wave function is:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[e^{i\theta(t)}\alpha + ie^{-i\theta(t)}\beta\right] ,$$

where α and β are eigenfunctions of S_z with eigenvalues $\pm\frac{1}{2}\hbar$, respectively, and $\theta(t)$ is a real function of time that you should determine explicitly.

(c) At a time $t > T$, we measure S_y. What are the possible results of this measurement and with what probabilities? Find a relation between ω_0 and T such that the measurement of S_y yields a unique result. [10 points]

Note: this problem was assigned as part of the UCSC Physics written qualifying exam for Quantum Mechanics.