Homework Set #4.

Due Date: Wednesday February 6, 2019

1. $\psi(x,t)$ is a solution of the Schrödinger equation for a free particle of mass m in one dimension, and

$$\psi(x,0) = A\exp(-x^2/a^2).$$

(a) At time $t=0$ find the probability amplitude in momentum space.

(b) Find $\psi(x,t)$.

2. For a simple harmonic oscillator, consider the set of *coherent states* defined as

$$|z\rangle \equiv e^{-|z|^2/2}\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{n!}}|n\rangle$$

in terms of the complex number z, where $|n\rangle$ is an eigenstate of the number operator $N = a^\dagger a$.

(a) Show that the coherent states are normalized to unity. Prove that they are eigenstates of the annihilation operator a with eigenvalue z.

(b) Calculate the expectation value $\mathcal{N} = \langle N\rangle$ and the uncertainty $\Delta\mathcal{N}$ in such a state. Show that in the limit $\mathcal{N}\to\infty$ of large occupation number the relative uncertainty $(\Delta\mathcal{N})/\mathcal{N}$ tends to zero.

(c) Suppose that at time $t=0$, the oscillator is in an eigenstate state of the annihilation operator a with eigenvalue z. Calculate the probability of finding the system in this state at later time $t>0$ as a function of z and t.

Note: this problem was assigned as part of the UCSC Physics written qualifying exam for Quantum Mechanics.