Homework Set #7.

Due Date: Wednesday March 6, 2019

1. Let $\vec{J}$ be an angular momentum operator.

   (a) Using the usual angular momentum commutation relations, prove that $\vec{J}^2 = J_z^2 + J_+ J_- - \hbar J_z$.

   (b) Using (a), derive the relation

   $$J_- \psi_{j,m} = c_- \psi_{j,m-1}.$$

2. A particle in a spherically symmetrical potential is known to be in an eigenstate of $\vec{L}^2$ and L_z with eigenvalues $\hbar^2 l(l+1)$ and $m\hbar$, respectively. Prove that the expectation values between $|lm>$ states satisfy

   $$< L_x >=< L_y >= 0, \qquad < L_x^2 >=< L_y^2 >= \frac{[l(l+1)\hbar^2 - m^2\hbar^2]]}{2}.$$

   Give a semiclassical interpretation of this result.

3. The wave function of a particle subjected to a spherically symmetric potential $V(r)$ is given by

   $$\psi(\vec{x}) = (x + y + 3z) f(r).$$

   (a) Is ψ an eigenfunction of $\vec{L}^2$? If so, what is the l-value? If not, what are the possible values of l we may obtain when $\vec{L}^2$ is measured?

   (b) What are the possibilities for the particle to be found in various m, l states?

   (c) Suppose it is known somehow that $\psi(\vec{x})$ is an energy eigenfunction with eigenvalue E. Indicate how we may determine $V(r)$.

4. Consider an electron in a uniform magnetic field along the z direction, with a Hamiltonian

   $$H = -\vec{\mu} \cdot \vec{B} = \hbar\omega\sigma_z, \qquad \omega = eB/2mc$$

   Let the result of a measurement be that the electron spin is along the positive y direction at $t = 0$. Find the Schrödinger state vector for the spin, and the average polarization (expectation value of S_x) along the x direction for $t > 0$.