Homework Set #8.

Due Date: Wednesday March 13, 2019

1. Consider a particle whose wave function is

$$\psi(x,y,z) = \frac{1}{4\sqrt{\pi}}\frac{2z^2 - x^2 - y^2}{r^2} + \sqrt{\frac{3}{\pi}}\frac{xz}{r^2}$$

   (a) What is the total angular momentum ℓ of the particle?

   (b) If a measurement of the z-component of the angular momentum were made, what possible measurements could result, and what is the probability for each?

   [*Winter 2015 UCSC Written Qualifying Exam*]

2. Consider an orbital angular momentum eignestate $|l = 2, m = 0>$. Suppose this state is rotated by an angle β about the y-axis. Find the probability for the new state to be found in $m = 0,\ \pm 1$ and ± 2.

3. We are to add angular momenta $j_1 = 1$ and $j_2 = 1$ to form $j = 2,\ 1$ and 0 states.

   (a) Using either the ladder operator method or the recursion relation, express all (nine) $\{j, m\}$ eigenstates in terms of $|j_1 j_2; m_1 m_2>$.

   (b) The $\rho^+,\ \rho^0,\ \rho^-$ mesons belong to an "iso-spin" $I = 1$ triplet, the three states corresponding to the three values of I_z. The same holds for the three pions. Using the results of part (a) above, and knowing that isospin is conserved in the decay, calculate the decay branching fraction of the neutral ρ^0 into a $\pi^0\pi^0$ final state.